THE DRYING OF FIBERS: A HEAT AND MASS TRANSFER MODEL

Juan Rodríguez-Ramírez[1], Pedro Quintana-Hernández[2], Lilia Méndez-Lagunas[1] and
Hugo Jiménez-Islas[2]

1. Centro Interdisciplinario de Investigación Para el Desarrollo Integral Regional, IPN,
Unidad Oaxaca, Calle Hornos 1003, Santa Cruz Xoxocotlán, Oax., México.
E-Mail: jrodrigr@redipn.ipn.mx
2. Instituto Tecnológico de Celaya, Av.. Tecnológico y García Cubas s/n. Celaya, Gto.,
México. E-Mail: pedro@iqcelaya.itc.mx

Key words: heat and mass transfer, modeling, sugar cane bagasse

ABSTRACT

A mechanistic model was used to describe the drying processes at different drying conditions. The model was discretized using double orthogonal collocation and solved with a Runge-Kutta-Fehlberg method. The values of heat and mass transfer coefficients and effective diffusivity obtained experimentally were used in the solution of the model. The numerical solution presented a deviation, respect to the experimental data, at the beginning of the drying process (during the conditioning period) but is followed well during the constant rate and falling rate periods. Further simulations showed that the drying process of fibers included the following mechanisms: evaporation, liquid diffusion and superficial effects.

INTRODUCTION

Mechanistic models usually describe the behavior of systems through the appropriate validation of process mechanisms. These mechanisms are evaluated independently and usually expressed with a set of coupled differential equations in time and space. The equations are obtained from mass and energy balances and constitutive equations took into consideration the characteristics of internal structure and material surface. The boundary and initial conditions are selected depending upon the operation drying conditions and initial material characteristics. There are two technical problems when solving mechanistic models (Kiranoudis et al., 1995). The first one is to find a explicit solution for the set of differential equations; and the second one is related to the use of non linear parameter. Both problems may be solved by numerical approximations.

In the first reported studies of drying, diffusional mechanisms were considered as the principal cause of moisture migration; but at present, it is well known that there are different internal transport mechanisms that control the drying process (Przesmycky and Strumillo, 1985). For example, for describing adequately the drying of biological materials, the mathematical models must consider simultaneous momentum; heat and mass transfer effects as well as the effect of variable diffusivity at each drying period.

Other approach for deriving drying models is based on the analysis of heat and mass transfer equations expressed in terms of volume averaged quantities (Thorpe and Whitaker, 1992). This approach is difficult to be implemented, but allows considerable detail to be retained. In this work, the model used for the analysis of drying parameters consisted of a set of coupled heat and mass transfer partial differential equations.

PROCEDURE

The model used in this paper takes into consideration the following assumptions: a) sugar cane fibers have cylindrical shape and are structurally homogeneous, b) effective values for diffusivity and thermal conductivity can be used, c) the drying process is analyzed without considering the particular effects of constant and falling rate periods, d) during the drying process, moisture content and solid temperature are the only forces for mass and heat transport, e) the mathematical model included the analysis in the radial direction only, f) the ratio between average radius and average length of the fibers is greater than 30, g) the diffusion coefficient D is a non linear function of both moisture content and temperature, and h) at the beginning, moisture content and temperature are constant along the fibers. The equations 1 and 2 represent the mass and heat balances in cylindrical coordinates and equation 3-8 represent the initial and boundary conditions:

$$\frac{\partial(\rho_p X_s)}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left[r \cdot \rho_p \cdot D \frac{\partial X_s}{\partial r}\right] \tag{1}$$

$$\frac{\partial(\rho_p h_s)}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left[r \cdot k \cdot \frac{\partial T_s}{\partial r}\right] \tag{2}$$

$$X_s = X_0 \qquad r_0 \geq r \geq 0 \qquad t = 0 \tag{3}$$

$$\frac{\partial X_s}{\partial r} = 0 \qquad r = 0 \qquad t \geq 0 \tag{4}$$

$$-\rho_p D \frac{\partial X_s}{\partial r} = h_M(a_{we} - a_w) \qquad r = r_0 \qquad t \geq 0 \tag{5}$$

$$T_s = T_0 \qquad r_0 \geq r \geq 0 \qquad t \geq 0 \tag{6}$$

$$\frac{\partial T_s}{\partial r} = 0 \qquad r = 0 \qquad t \geq 0 \tag{7}$$

$$k \frac{\partial T_s}{\partial r} = h_H(T_a - T_s) - \Delta H_s \, h_M(a_{we} - a_w) \qquad r = r_0 \qquad t \geq 0 \tag{8}$$

Ecuation 9 shows the effective diffusiviy as a function of temperature and moisture content.

$$D = D_0 X_s^{E_X} \exp(-E_T / RT_s) \tag{9}$$

The proposed model involves mass and heat transfer governed by internal and external mechanisms. The corresponding parameters of the model for mass transfer are the effective diffusivity and the convective coefficient of mass transfer; for heat transfer, the parameters are thermal conductivity and external heat transfer coefficient. The state variables of the system are temperature and solid moisture content. The specific enthalpy of the fibers is a linear function of the moisture content of the material and temperature and it is given by equation (10).

$$h_s = cp_s T_s + X_s cp_w T_s \tag{10}$$

Recio et al. (1987) determined the heat capacity of sugar cane bagasse as a function of the temperature and found a variation of less than 10% in the interval from 40 to 110 °C. In this work, the average value is used. The same consideration was made for the heat capacity of water. The apparent density of the solid phase was calculated based on the total volume of particles, including pores (equation 11). The size of fibers was considered constant during the drying process.

$$\rho_p = \rho_s \cdot (1-\varepsilon) \tag{11}$$

Water activity in the gas phase, a_{we}, is a function of the moisture content and temperature. It is determined by solving equation the 12 (GAB equation).

$$X_{se} = \frac{X_M \cdot C \cdot K \cdot a_{we}}{(1 - K \cdot a_{we})(1 - K \cdot a_{we} + C \cdot K \cdot a_{we})} \tag{12}$$

The partial differential equations were solved by spatial discretization with double orthogonal collocation (Jiménez and López, 1996). The new set was generated with ordinary differential equations, and it was solved by integration using the explicit method of Runge-Kutta-Fehlberg. The moisture content at each point was integrated in the space domain to obtain an averaged value. The calculated values were compared with the experimental values obtained by Quintana et al., (1997). Rodríguez, 1998. Moistened Sugar cane bagasse fibers were dried in a thermogravimetric Analyzer Stanton Redcroft STA 780. Four different air temperatures, two-air flow levels (0.35, 0.8 cm/s) and two sizes of fibers (0.36mmx7.5mm, 0.55mmx10.2mm). Surface Fiber temperature and weight were measured. From this information, experimental heat and mass transfer coefficients and effective diffusion were calculated.

An analysis of sensibility was done for heat and mass transfer coefficients and diffusivity coefficient in order to find the parameter's best values.

RESULTS

The model parameters (h_M, h_H, D_0, y E_T) were calculated by testing the experimental model and experimental values. The parameter effect was considered analyzing the temperature and moister content graphics of fibers (sensibility analysis). The numerical model solution and the experimental data were matched so the determinant of mean sum of the squares of material content and temperature be smallest than 0.004 (Kiranoudis et al., 1995)

Figure 1 shows the dimensionless values for moisture content and temperature. The calculated values for the temperature of the solid deviate considerable at the final of the drying process (dimensionless time greater than 800). Deviation in moisture content was less than 2%. In order to analyze the effect of heat and mass transfer coefficients, their values were reduced by 15% and plot against the experimental data.

Figure 2 shows the solution of the model when the reduced values were used. A reduction in the heat transfer coefficient, h_H, generated small values for the solid temperature, in the constant rate period, as well as for the drying rate. The same analysis for the mass transfer coefficient, h_M, shows a decrement in the drying rate and solid temperature at the falling rate period. Figure 3 shows the effects of decreasing the diffusivity coefficient. The analysis was made changing D_0 and E_T. Only the first parameter had a great impact on drying rate and solid temperature at the falling rate period. The Ex parameter was a constant value defined by a regression value of diffusivity versus moisture content (Rodriguez, et al., 2000).

Figure 4 depicts the concentration profiles with relation to the radial coordinate at different times. At the beginning, the moisture content was uniform inside the fiber. There were not profiles with large variations throughout the radial coordinate. Moisture content at the surface diminished progressively in the way of an evaporative front. The analysis of sensibility confirmed the validity of the assumptions made to solve the model. Internal resistance due to the diffusion was one of the principal factors that affected the drying of fibers. In addition, it was found that the effects of the external heat transfer must be taken into consideration when solving the model.

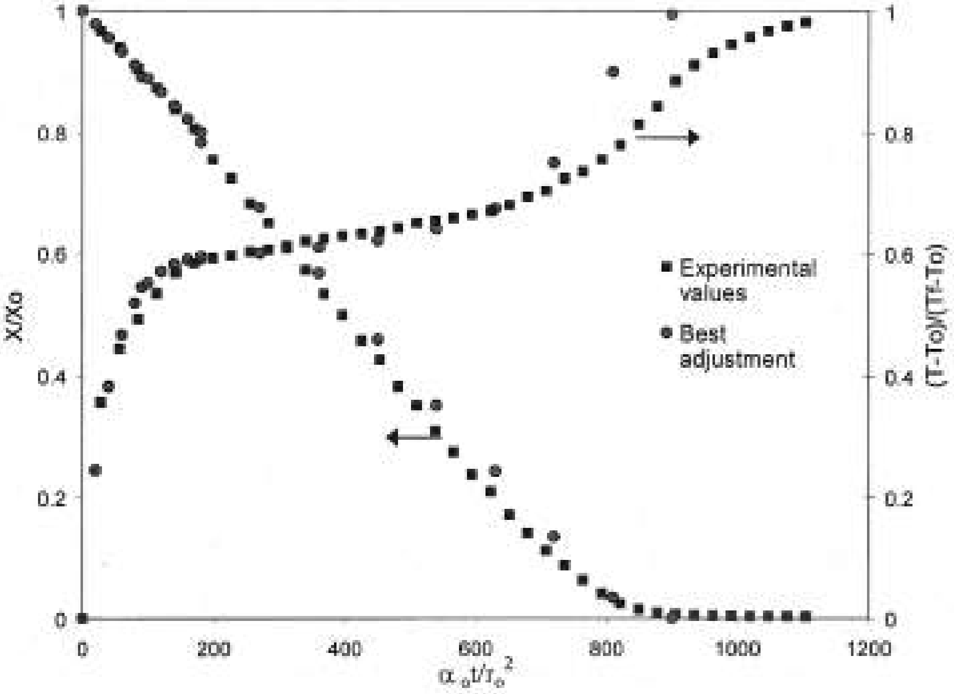

Figure 1. Comparison of moisture content and temperature for adjusted model and experimental values.

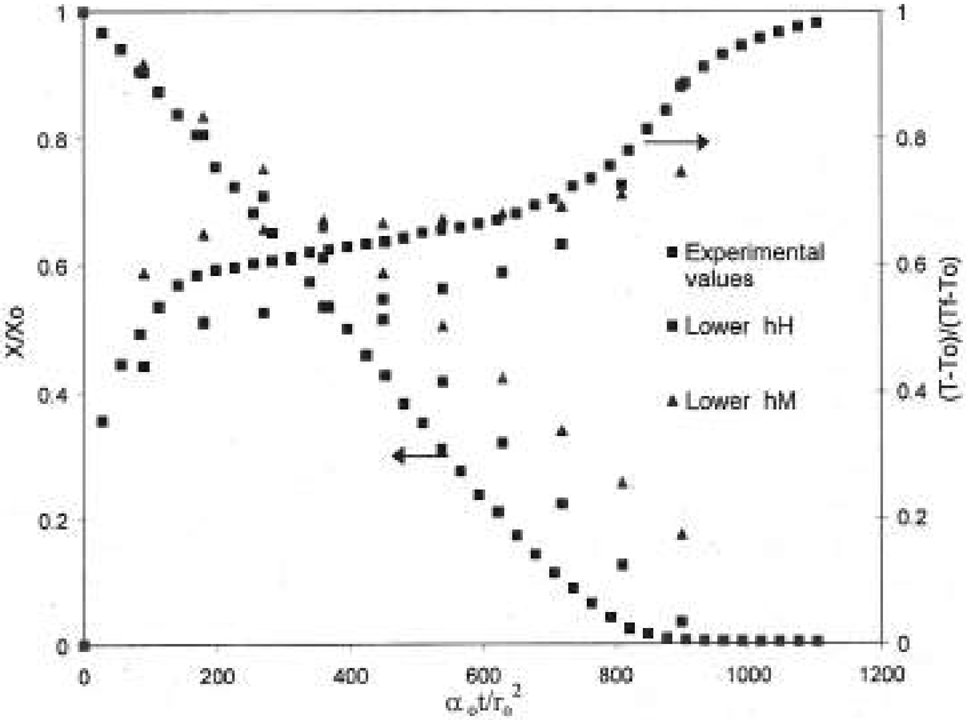

Figure 2. Effects of decreasing hH and hM (15%) on moisture content and temperature

The model predicts adequately the moisture content and solid temperature profiles up to a limit of 0.5 kg water /kg dried solid. At the end of the process, the model predicts a higher solid temperature than the experimental values. For practical purposes, sugar cane bagasse is rarely dried beyond 0.5 kg water/kg dried solid; therefore, the model may be used for the designing and simulation of dryers.

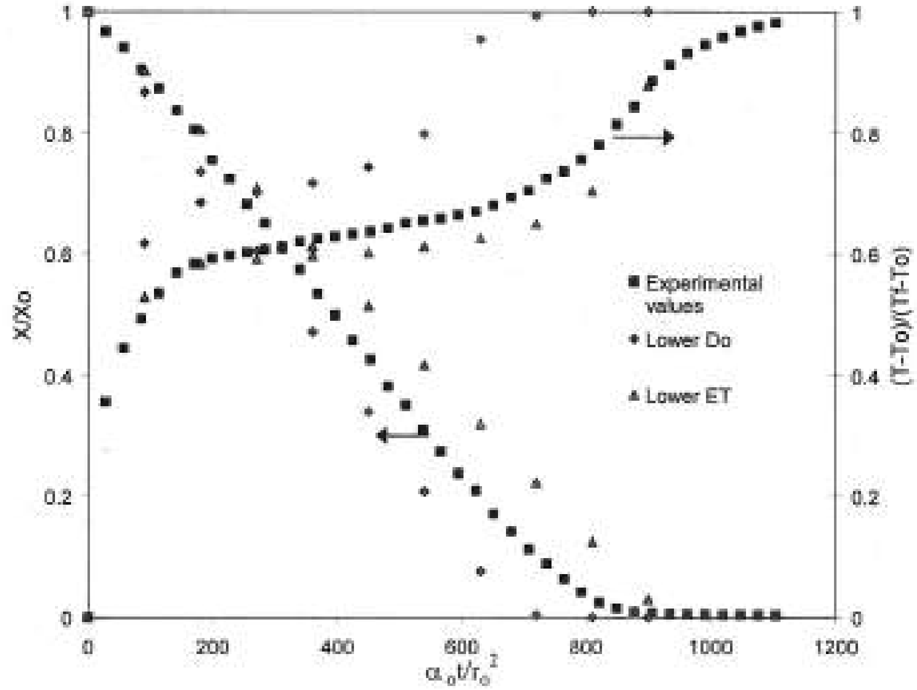

Figure 3. Effects of decreasing D_0 and E_T (15%) on moisture content and temperature.

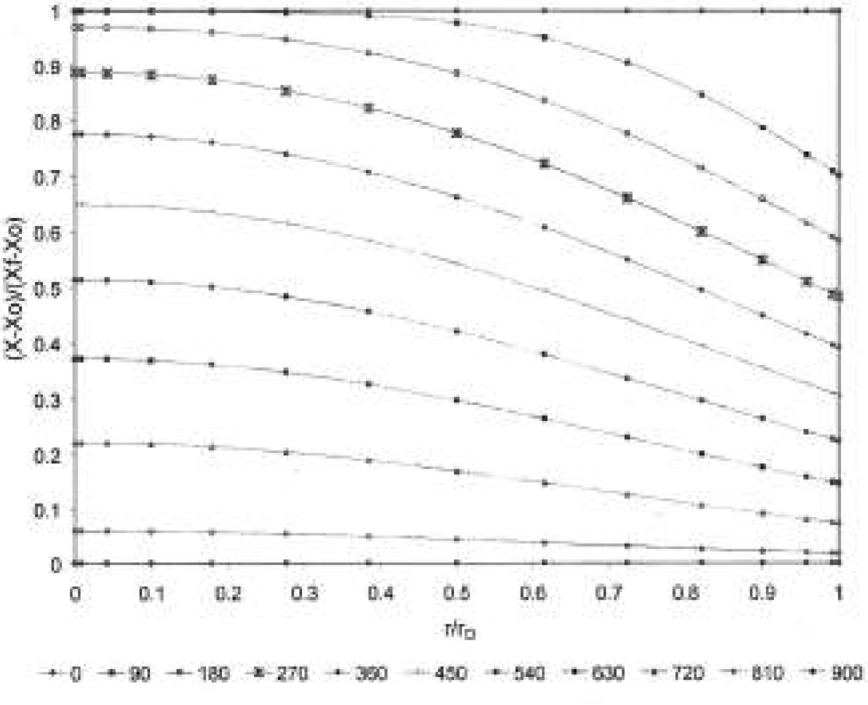

Figure 4. Moisture content profiles for sugar cane bagasse at different dimensioneless times $\alpha_0 t/r_0^2$.

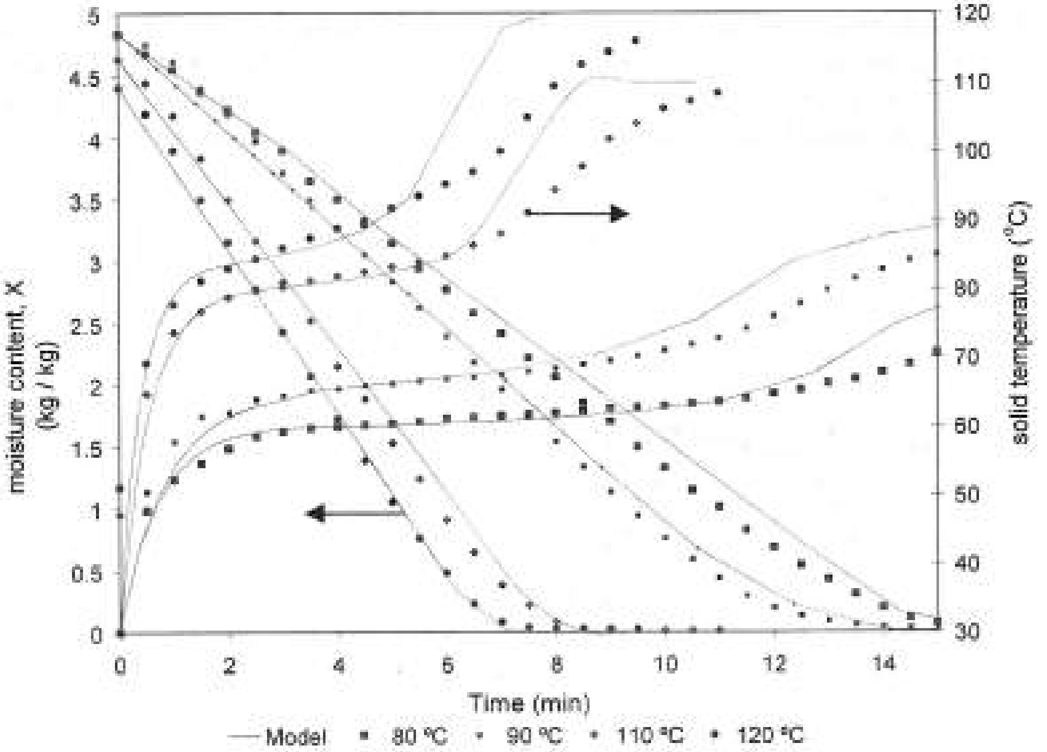

Figure 5. Experimental and calculated values at different drying temperatures.

Table I shows the adjusted parameters used by the model and the experimental values obtained from the experimental drying curves. Adjusted convective transport coefficients have the same order of magnitude that those calculated experimentally at 80 and 90 °C. At higher temperatures, the difference between the model and experimental coefficients increase. Therefore, the model can be used at temperatures under the water boiling point without any modification of the model.

Table I. Experimental parameters and estimates of the proposed mathematical model

Temp. °C	h_H (W/m^2-K)		h_M (kg/m^2-s)		D_O (10^{-5} m^2/s)		E_T (kJ/mole)	
	Exp.	Model	Exp.	Model	Exp.	Model	Exp.	Model
80	41.34	29.92	0.04	0.08	0.12	0.121	27.02	24.31
90	46.32	31.37	0.05	0.10	0.12	0.156	27.02	25.67
110	40.53	43.75	0.04	0.18	0.12	0.188	27.02	22.68
120	46.92	58.72	0.05	0.26	0.12	0.232	27.02	24.31
130	53.45	59.54	0.05	0.27	0.12	0.243	27.02	20.03

CONCLUSIONS

The drying of fibers may be analyzed through a series of mechanisms (evaporation, liquid diffusion, surface effects, etc.) or solving a global model with adjusted parameters. Internal resistance due to the liquid diffusion is one of the principal mechanisms of mass transportation during the drying of sugar cane bagasse fibers. The solved model included parameters for external and internal heat and mass transfer resistance (heat transfer coefficient, thermal effective conductivity, effective diffusivity and mass transfer coefficient), and an equation for the diffusivity as a function of temperature. Finally, the proposed model simulates adequately the drying process of sugar cane bagasse fibers when air temperature is lower than the water boiling. For greater temperatures, it should be necessary to determine the parameters that would describe adequately the experimental tests or even to modify the model.

ACKNOWLEDGMENT

The authors wish to acknowledgment the support of this research by CONACyT under grant number I289228, CGPI-IPN under grant number 990390 and COFAA-IPN, whose assistant is hereby gratefully.

NOTATION

a_w	air humidity	.../...
a_{we}	equilibrium humidity	.../...
C	parameter in the GAB equation	.../...
cp_s	solid heat capacity	J/kg K
cp_w	water heat capacity	J/kg K
D	diffusion coefficient	m^2/s
D_0	parameter in equation 9	m^2/s
E_T	parameter in equation 9	kJ/mole K
E_X	parameter in equation 9	.../...
h_H	external heat transfer coefficient	W/m^2 K
h_M	mass transfer coefficient	kg/m^2
h_s	specific enthalpy of the solid	J/kg
k	thermal conductivity	W/m K
K	parameter in the GAB equation	.../...
R	radial coordinate	m
r_0	Radius of the fiber	m
T_s	solid temperature	K
T_0	initial solid temperature	K
t	time	s
X	moisture content, dry basis	kg/kg
X_M	parameter in the GAB equation	.../...
X_s	solid moisture content, dry basis	kg/kg
X_0	solid initial moisture content, dry basis	kg/kg
Greek Symbols		
ΔH_s	latent heat	J/kg
ε	porosity, $(1-\rho_p/\rho_s)$	.../...
ρ_p	apparent density of solid, including pores	kg dried $solid/m^3$
ρ_s	real density excluding pores	kg dried $solid/m^3$

LITERATURE

Jiménez, I. H., and López, I. F., (1996), Parcol2: Programa para resolver EDP parabólicas bidimensionales no lineales, por doble colocación ortogonal, Avances en Ingeniería Química, Vol. 6, no. 2, pp. 168-173

Kiranoudis C.T., Maroulis Z.B., and Marinos-Kouris, D., (1995), Heat and Mass Transfer model building in drying with multiresponse data, Int. J. Heat Mass Transfer, Vol. 38, no. 3, pp. 463-480

Méndez L.L., (1998), Estudios sobre el secado de bagazo de caña, Tesis de Maestría en Ciencias en Ing. Química, Instituto Tecnológico de Celaya.

Przesmycky, Z. y Strumillo, C., (1985), The mathematical modeling of drying process based on moisture transfer mechanism, Drying Technology, pp. 129-134

Quintana-H.P. A., Rodríguez-R. J., Méndez-L, L. L., and Cornejo-S. L., (1997), Humidity Diffusion within sugarcane fibers, Proc. of the Inter-American Drying Conference, IADC, Vol. B, pp. 538-542

Recio R. A., Oliva R. L. y Chestakou S., (1987), Calor específico del bagazo para temperaturas entre 303 y 323 K, Ing. Energetica, Vol. 8, no. 3, pp. 209-214

Rodríguez R. J., (1998), Modelamiento del Transporte de Humedad en Bagazo de Caña, Tesis de Doctorado en Ciencias en Ingeniería Química, Instituto Tecnológico de Celaya.

Rodríguez R.J., Quintana H.P.A., Méndez L. L.L.,Martínez G.G., Alatorre G.G., (2000), Diffusion of moisture in Drying of sugar cane fibers and bundles, Drying Tech., 18 (4), pp. 1091-1098.

Thorpe, G. R., and Whitaker, S., (1992), Local mass and thermal equilibria in ventilated grain bulks. Part I: The development of heat and mass conservation equations, J. Stored Prod. Res., Vol. 28, no. 1, pp. 15-17

Proceedings of the Second Inter-American Drying Conference

Boca del Rio, Veracruz, Mexico, July 8-10, 2001

Editor Krzysztof N. Waliszewski

Published by Instituto Tecnológico de Veracruz,
M.A. de Quevedo 2779, 91860 Veracruz, Ver., Mexico

ISBN 968-5401-01-2

Printed by Editorial Ducere SA de CV, Rosa Esmeralda 3 Bis, Col.
Molino Rosas, Del. Gustavo Obregon, México D.F., México

Simultaneous heat and mass diffusion including shrinkage during drying of banana: a numerical simulation 331
A. G. B. de Lima, S. A. Nebra and M. R. de Queiroz, Federal University of Paraiba, Campina Grande-PB, Brasil

Direct drying of foods: contamination aspects 343
Marzouk Benali, CANMET, Varennes, Quebec, Canada

Thermal processing of hog manure in a continuous multistage screw-in-trough dryer 353
Marzouk Benali and Tadeusz Kudra, , CANMET, Varennes, Quebec, Canada

Osmodehydration of apple slices in solutions of sucrose and natural polymers 359
Ebner Azuara and César J. Beristain, Universidad Veracruzana, Xalapa, Veracruz, México

Section III. Wood and paper

Reduction of defects in the drying of red oak at atmospherics conditions 367
Augustin Pérez Ricárdez, Luis Jorge Aviña Berumen, Ricardo Andrade Ramirez and Akira Tomita, Instituto Tecnológico de Durango, Durango, Durango, México

Durango red oak drying at low temperature 371
José Arturo Pérez Galindo and Ignacio Martin Dominguez, Instituto Tecnológico de Durango, Durango, Durango, México

Drying of fibers : a heat and mass transfer model 379
Juan Rodríguez-Ramírez, Pedro Quintana-Hernández, Lilia Méndez-Lagunas and Hugo Jiménez-Islas, Centro Interdisciplinario de Investigación para el Desarrollo Integral Regional del Instituto Politécnico Nacional, Santa Cruz Xoxocotlán, Oax., México

Changes in swelling and tangential compression strength of mahogany wood influenced by sorption history 387
Rosven L.Arévalo and Roger E.Hernández, Université Laval, Laval, Quebec, Canada

Bacterial treatment of *Pinus radiata* timber to improve permeability 395
Justin J. Nijdam, E.Lehmann and R.B.Keey, University of Canterbury, Canterbury, New Zealand

Experimental study of drying and residence time of sugar cane bagasse in cyclonic devices 407
Jefferson L.G. Corréa, Marcio O. Chamma, Adolfo L. Godoy, Maria A. Silva and Silvia A. Nebra , State university of Campinas, Campinas, SP, Brasil